International Journal of Public Administration and Management Research (IJPAMR), Vol. 3, No. 3, May, 2016.
Website: http://www.rcmss.com. ISSN: 2350-2231 (Online) ISSN: 2346-7215 (Print)
Emeje, M. A. & Onalo, S. E. , 2016, 3(3):80-88

TABULAR METHOD OF INTEGRATION BY PARTS AND SOME OF ITS STRIKING APPLICATIONS

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ABSTRACT

We study one of the celebrated methods of integration by parts “the Tabular Method”. We investigate some of the less familiar applications. This method is a valuable tool for finding integrals and can be applied to more advanced topics including the derivations of some important theorems in Mathematical analysis.

Keywords: *Alternate signs, Domain, Laplace Transform, Method.*

INTRODUCTION

In mathematical analysis, **integration by parts** is a theorem that relates the integral of a product of functions to the integral of their derivative and anti derivative. It is frequently used to transform the anti derivative of a product of functions into an anti derivative for which a solution can easily be found. The rule can be derived by integrating the product rule of differentiation.

If $u = u(x)$ and $du = u^{I}(x)\,dx$,

and $v = v(x)$ and $dv = v^{I}(x)\,dx$,

then integration by parts states that:

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int v(x)\,u'(x)dx$$

or more compactly:

$$\int u\,dv = uv - \int v\,du. \qquad \text{……………………(1)}$$

Example

To calculate

$$I = \int x\cos(x)\ dx\ ,$$

let:

$$u = x \ \Rightarrow\ du = dx$$
$$dv = \cos(x)\ dx \ \Rightarrow\ v = \int \cos(x)\ dx = \sin(x)$$

then:

$$\begin{aligned}\int x\cos(x)\,dx &= \int u\ dv\\ &= u\cdot v - \int v\,du\\ &= x\sin(x) - \int \sin(x)\ dx\\ &= x\sin(x) + \cos(x) + C,\end{aligned}$$

where *C* is a constant of integration.

For higher powers of *x* in the form

$$\int x^n e^x \, dx, \quad \int x^n \sin(x) \, dx, \quad \int x^n \cos(x) \, dx \; ,$$

repeatedly using integration by parts can evaluate integrals such as these; each application of the theorem lowers the power of *x* by one. The process can be lengthy and may required serious algebraic details as it will involves repeated iteration. Tabular method of integration by parts seems to offer solution to this problem.

TABULAR METHOD OF INTEGRATION BY PARTS

In problems involving repeated applications of integration by parts, a tabular method is more useful and can help to organize the work. This method will work to avoid tedious algebraic details that may come up as a result of repeated iterations.

Method

This method works well for integrals of the form

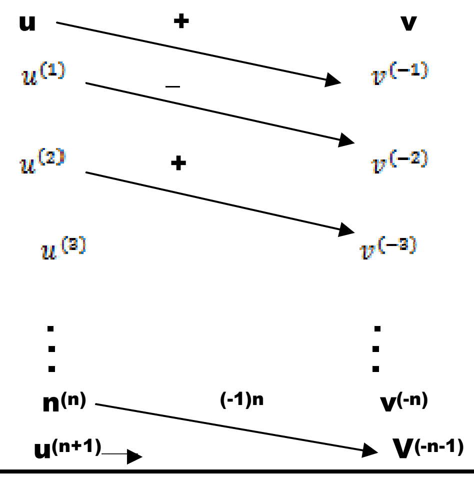
$$\int f(x) g(x) dx,$$

Where f is a polynomial and g is a function that can be integrated repeatedly.
This method can be demonstrated in several ways [1] .
The idea is simple: we take u = f (x) and dv = g(x) dx in our integration by parts. So then

u = f (x) and dv = g(x) dx
du = $f^{I}(x)$ dx and v = $\int g(x) dx$

then the integral is just $\int u dv$ **= uv −** $\int v du$**.**

U and its derivatives	V and its anti derivatives



This form is especially useful when $u^{(n)}$ becomes zero for some n (and, in particular, when u is a polynomial function with degree smaller than n). Hence, the integral evaluation can stop once the $v^{(n-1)}$ term has been reached.

The middle column switches ± signs, the first column differentiates u, and the second column anti differentiates dv. We can write the result of integration as

$$\int udv = uv - \int vdu$$

where u(x) is a polynomial and dv is a function that can be easily integrated repeatedly. Just take derivatives of u down to 0, and integrate dv just as many times. Now draw arrows as above across both sides (since we got to 0, the last, leftward pointing arrow can be omitted). Alternate signs on the arrows, starting from +. Now multiply across arrows and add together.

Now we write

$$\int u(t)v(t)dt = uv^{(-1)} - u^{(1)}v^{(-2)} + u^{(2)}v^{(-3)} - u^{(3)}v^{(-4)} + \ldots$$

$$+ (-1)^{n+1} \int u^{(n+1)} v^{(-n-1)}(t)\, dt$$

$$= \sum_{k=0}^{n} (-1)^K u^{(k)} v^{(-k-1)} +$$

$$(-1)^{n+1} \int u^{(n+1)}(t) v^{(-n-1)}(t) dt$$

Applications

problem 1

$$\int 3x^2\sqrt{x-1}\ dx$$

Let $u = 3x^2$ and $dv = \sqrt{(x-1)}\ dx$

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Using the Tabular Method

Sign	Derivatives	Integrals
+	$3x^2$	$(x-1)^{\frac{1}{2}}$
–	$6x$	$\frac{2}{3}(x-1)^{\frac{3}{2}}$
+	6	$\frac{4}{15}(x-1)^{\frac{5}{2}}$
–	0	$\frac{8}{105}(x-1)^{\frac{7}{2}}$

Hence the solution can be written as

$$\int 3x^2\sqrt{(x-1)}\,dx = 2x^2(x-1)^{\frac{3}{2}} - \frac{24x}{15}(x-1)^{\frac{5}{2}} + \frac{16}{35}(x-1)^{\frac{7}{2}}$$

+ C

Problem 2

consider the integral

$$\int x^3 \cos x\, dx.$$

Let $u = x^3$. Begin with this function and list in a column all the subsequent derivatives until zero is reached. Secondly, begin with the function d*v* (in this case cos (*x*)) and list each integral of d*v* until the size of the column is the same as that of *u*. The result should appear as follows.

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Derivatives of *u*	**Integrals of d*v***
x^3 +	$\cos x$
$3x^2$ −	$\sin x$
$6x$ +	$-\cos x$
6 −	$-\sin x$
0	$\cos x$

Now the **alternating** signs (beginning with the positive sign). Do so until further pairing leads to sums of zeros. The result is the following (notice the alternating signs in each term):

$$(+)(x^3)(\sin x) - (3x^2)(-\cos x) + (6x)(-\sin x) - (6)(\cos x) + C.$$

Which leads to the result

$$x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6 \cos x + C.$$

With proper understanding of the tabular method, it can be extended.

Problem 3

Consider

$$\int e^x \cos x \, dx.$$

Derivatives of *u*	**Integrals of d*v***
e^x	$\cos x$
e^x	$\sin x$
e^x --------	$-\cos x$

In this case in the last step it is necessary to integrate the product of the two bottom cells obtaining:

$$\int e^x \cos x \, dx = e^x \sin x + e^x \cos x - \int e^x \cos x \, dx,$$

which leads to

$$2 \int e^x \cos x \, dx = e^x \sin x + e^x \cos x,$$

$$\int e^x \cos x \, dx = \frac{e^x(\sin x + \cos x)}{2} + C.$$

Tabular Method of Integration and Laplace Transform

Application of Tabular method of Integration to the establishment of the fundamental formula for the Laplace transform of the nth derivative of a function.

Basic Definitions

Let *f*(*t*) be a function defined on $[0, \infty)$. The Laplace transform of *f*(*t*) is a new function defined as

$$\mathcal{L}(f)(s) = \int_0^\infty e^{-st} f(t) dt = \lim_{b \to \infty} \int_0^b e^{-st} f(t) dt.$$

The domain of $\mathcal{L}(f)$ is the set of $s \in \mathbf{R}$, such that the improper integral converges. We will say that the function *f* (*t*) has an **exponential order** at infinity if, and only if, there exist α and *M* such that

$$|f(t)| \leq M e^{\alpha t}, \quad \forall t \geq 0.$$

Suppose that *f*(*t*), and its derivatives $f^{(k)}(t)$, for $k \in [1, n]$, are piecewise continuous and have an exponential order at infinity. Then we have

$$\mathcal{L}(f^{(n)}) = s^n \mathcal{L}(f) - s^{n-1} f(0) - s^{n-2} f'(0) - \ldots - f^{(n-1)}(0).$$

This is a very important formula because of its use in differential equations. We shall use the Tabular method to establish this formula.

Prove of formula for Laplace transforms.

Proof

$$\mathcal{L}(f)(s) = \int_0^\infty e^{-st} f(t) dt = \lim_{b \to \infty} \int_0^b e^{-st} f(t) dt.$$

Put u= e^{-st} $and\ dv = f^{n}(t)dt$

e^{-st}	+	$f^{n}(t)$
-se^{-st}	−	$f^{(n-1)}(t)$
S$^2 e^{-st}$		$f^{(n-2)}(t)$
⋮		⋮
$(-1)^{n-1}s^{n-1}e^{-st}$	$(-1)^{n-1}$	$f^{I}(t)$
$(-1)^{n}s^{n}e^{-st}$ ------	$(-1)^{n}$	$f(t)$

$$\mathcal{L}(f)(s) = \int_0^{\infty} e^{-st} f(t)dt = \lim_{b\to\infty} \int_0^{b} e^{-st} f(t)dt.$$

$$= [e^{-st}f^{(n-1)}(t) + se^{-st}f^{(n-2)}(t) + s^2e^{-st}f^{(n-3)}(t) + \cdots + s^{n-1}e^{-st}f(t)$$

$$]_{t=0}^{t=\infty} + s^n \int_0^{\infty} e^{-st}\ f(t)$$

$$\mathcal{L}(f^{(n)}) = s^n\mathcal{L}(f) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots. - f^{(n-1)}(0).$$

APPLICATION OF TABULAR METHOD OF INTEGRATION: A RECIPE FOR SUSTAINABLE DEVELOPMENT IN NIGERIA

Brundland Commission's Report (1987) sees Sustainable Development as the process of change in the exploitation of resources and creation of the social,economic,technological development and ensuring that they are in harmony.Whereas,Adikwu (2007) posits that Sustainable development is the all - round growth which peameates the physical,social,economic and political attributes of human endavours.Moreso, Ojih (1996) asserts that Sustainabke development is seen in relation to man's acton targeted at applying an increasing level of efficient technology with the view to controlling natural resources in order to broaden the level of growth of national output as well as income per Capital for a viable and reliable economic disposition. From these conceptualizations, Sustainable development seeks to meet the needs and aspirations of the citizens at the present without compromising the capability to meet future needs (Tade , Ademola 1992).

From the above discourse, accurately accomplishing sustainable development is optimally applicable through a suitable methodology of tabular method of integration. Integration and derivatives are also very significance to this discourse knowingfulwell the pivotal role tabular method of integraton plays to national development vis-a-vis sustainable development.

The application of tabular method of integration play a sigificance role through serving as an instrumentality for guilding economic decisions, cost control mechanism, policy formulation /governance,proper and reliable weather forecast which could aid agriculture,health care services through policy initiation and implementation,modelling and manufacturing.

Therefore, considering the glaring evidences of low per capital income,revenue and expenditure gap,unemployment,low level of science and technological innovation, the application of tabular method of integration would serve as recipe for sound economic emancipation and Nigerian sustainable development since mathematics is a way of applying careful reasoning to solving man's problems in the society.

DISCUSSION

The great advantage of using the tabular method is the ability to do integration by parts *mechanically*, without needing to exert a large amount of thinking about the special circumstances of the problem. The procedure is fairly quick to memorize and easy to retain. After you learn it once, it will always be at your disposal as a tool for quickly and easily determining indefinite integrals that would otherwise take an immense amount of time to find.

In applications, problem 3.1 is a case where one of the functions is raised to fractional power. Our choice of u= x^3 is appropriate to enable one of the derivatives u^n to hit zero. It is much easier to apply the tabular method. In Problem 3.2 , both functions represented by u and dv are “smooth” enough to allow repeated differentiation and integration, respectively. Problem 3.3 has another dimension where none of the derivatives of u is zero. This problem is solved using tabular method and the solution obtained by an algebraic operation.

Problem 4.2 is a case where Tabular method is used to establish the fundamental formula for the Laplace transform of the nth derivative of a function.

Thanks to the work of David Horowitz (The College Mathematics Journal, September 1990, Volume 21,Number 4,pp. 307-311) where it is shown that Tabular integration by parts provides a straight forward proof of Taylor’s formula with integral remainder term and also applicable to complex line integrals. The List of applications in theorems of modern Mathematics is endless.

CONCLUSION

This research concludes that tabular method of integration by parts has significant impact to solving salient issues bordering human society and could be applied and harnessed towards advancing the course of human endavours and sustainable development.

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